DEPARTMENT OF PHYSICS, VNIT NAGPUR

PHL-516-MATHEMATICAL PHYSICS –AUTUMN 2019

Assignment 2, Due date: 21 August, 2019 (5:30 pm)

NOTE: (i) Assignments should be completed individually.

(ii) Write your solutions clearly, showing all steps.

	SECOND ORDER ODEs, Fourier transforms, Laplace transforms
Q.1	A block of wood is floating in the water; it is depressed slightly and then released to oscillate up and down. Assume that the top and bottom of the block are parallel planes which remain horizontal during the oscillations and that the sides of the block are vertical. Show that the period of the oscillation is $2\pi\sqrt{h/g}$ where *h* is the vertical height of the part of the block under water when it is floating at rest.
Q.2	The differential equation of a hanging chain supported at its end is $y''^2 = k^2(1+y'^2)$. Solve the equation to find the shape of the chain.
Q.3	The curvature of a curve in the (x,y) plane is $K = y''(1+y'^2)^{-3/2}$ with K=constant, solve this differential equation to show that curves of constant curvature are circles (or straight lines).
Q.4	The natural period of an undamped system is 3 sec, but with a damping force proportional to the velocity, the period becomes 5 sec. Find the differential equation of motion of the system and its solution.
Q.5	A mass *m* falls under gravity (force mg) through a liquid whose viscosity is decreasing so that the retarding force is *-2mv/(1+t*), where *v* is the speed of *m*. If the mass starts from rest, find its speed, and how far it has fallen (in terms of *g*) when *t*=1.
Q.6	The differential equation for the path of a planet around the sun (or any object in an inverse square force field) is, in polar coordinates, $\frac{1}{r^2}\frac{d}{d\theta}\left(\frac{1}{r^2}\frac{dr}{d\theta}\right)-\frac{1}{r^3}=-\frac{k}{r^2}$. Make the substitution u=1/r and solve the equation to show that the path is a conic section.
Q.7	Find the Fourier transform of $H(x-a)e^{-bx}$ where *H*(*x*) is the Heaviside function.
Q.8	By taking the Fourier transform of the equation $\frac{d^2\phi}{dx^2}-K^2\phi = f(x)$ show that its solution, *φ*(*x*), can be written as $\phi(x)=\frac{-1}{2\pi}\int_{-\infty}^{\infty}\frac{e^{ikx}\tilde{f}(k)}{k^2+K^2}dk$ where $\tilde{f}(k)$ is the Fourier transform of *f*(*x*).
Q.9	Find Laplace transform of L1-L17 page 469, ML Boas.
Q.10	Evaluate the following integrals by using the Laplace transform table: (i) $\int_0^{\infty} e^{-2t}\sin 3t\,dt$ (ii) $\int_0^{\infty}\frac{1}{t}e^{-2t}\sin(t\sqrt{2})dt$

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