DEPARTMENT OF PHYSICS, VNIT NAGPUR

PHL-516-MATHEMATICAL PHYSICS –AUTUMN 2019

Assignment 1, Due date: 2 August, 2019 (5:30 pm)

NOTE: (i) Assignments should be completed individually.

(ii) Write your solutions clearly, showing all steps.

FIRST ORDER ODEs

Q.1 The momentum *p* of an electron at speed *v* near the speed *c* of light increases according to the formula $p = \frac{mv}{\sqrt{1 - v^2/c^2}}$ where *m* is the constant (mass of electron). If an electron is subject to a constant force *F*, Newton's second law describing the motion is $\frac{dp}{dt} = \frac{d}{dt}\frac{mv}{\sqrt{1 - v^2/c^2}} = F$. Find v(t) and show that $v \to c$ as $t \to \infty$. Find the distance travelled by the electron in time t if it starts from rest.

Q.2 A mass *m* is accelerated by a time-varying force $\alpha \exp(-\beta t)v^3$, where *v* is its velocity. It also experiences a resistive force *ηv*, where *η* is a constant, owing to its motion through the air. The equation of motion of the mass is therefore $m\frac{dv}{dt} = \alpha \exp(-\beta t)v^3 - \eta v$. Find an expression for the velocity *v* of the mass as a function of time, given that it has an initial velocity v_0.

Q.3 A reflecting mirror is made in the shape of the surface of revolution generated by revolving the curve *y*(*x*) about the *x*-axis. In order that light rays emitted from a point source at the origin are reflected back parallel to the *x*-axis, the curve *y*(*x*) must obey, $\frac{y}{x} = \frac{2p}{1 - p^2}$ where *p* = *dy/dx*. By solving this equation for *x*, find the curve *y*(*x*).

Q.4 Heat is escaping at a constant rate (dQ/dt) through the walls of a spherical cavity. Find the temperature T at a distance *r* from the axis of the cylinder if the inside wall has radius *r*=1 and temperature T=100 and the outside wall has *r*=2 and *T*=0.

Q.5 Show that the thickness of the ice on lake increases with the square root of the time in cold weather, making the following simplifying assumptions. Let the water temperature be a constant 10˚ C, the air temperature a constant -10˚, and assume that at any given time the ice forms a slab of uniform thickness x. The rate of formation of ice is proportional to the rate at which heat is transferred from water to the air. Let *t=0* when *x=0*.

Q.6 According to the Newton's law of cooling, the rate at which the temperature of an object changes is proportional to the difference between its temperature and that of its surroundings. A cup of coffee at 200˚ is a room of temperature 70˚ is stirred continually and reaches 100˚ after 10 minutes. At what time was it at 120˚?

Q.7 Extend the radioactive decay problem one more stage, that is, let λ_3 be the decay constant of polonium and find how much polonium is there at time t.

Q.8 Find the general solution of $L\frac{dI}{dt} + RI + q/C = V$ for an RL circuit with $V=V_0 \cos\omega t$.

Q.9 Find the general solution of $L\frac{dI}{dt} + RI + q/C = V$ for an RC circuit with $V=V_0 \cos\omega t$.

Q.10 By finding an appropriate IF solve the equation $\frac{dy}{dx} = -\frac{2x^2 + y^2 + x}{xy}$

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