DEPARTMENT OF PHYSICS, VNIT NAGPUR

PHL-516-MATHEMATICAL PHYSICS –AUTUMN 2019

Assignment 3

NOTE: (i) Assignments should be completed individually.

(ii) Write your solutions clearly, showing all steps.

	Functions of Complex variable, Linear Vector space
Q.1	Find the Laurent series for the following functions about the indicated points; hence find the residue of the function at the point: (a) $\frac{1+\cos z}{(z-\pi)^2},\ z=\pi$ (b) $\frac{1}{z^2-5z+6},\ z=2$
Q.2	Find the residues of the following functions at the indicated points (without finding Laurent series): (a) $\frac{e^{2\pi iz}}{1-z^3}\ at\ z=e^{2\pi i/3}$ (b) $\frac{z-2}{z^2(1-2z)^2},\ z=0\ and\ z=\frac{1}{2}$
Q.3	Evaluate the contour integral: $\int_0^\infty \frac{x^2}{x^4+16}dx$
Q.4	Evaluate the contour integral: $\int_0^\infty \frac{\cos\pi x}{1+x^2+x^4}dx$
Q.5	Evaluate the contour integral: $\int_{-\infty}^\infty \frac{x}{(x-1)^4-1}dx$
Q.6	For the given set of basis vectors, use the Gram-Schmidt method to find an orthonormal set: (a) A = (0,2,0,0); B=(3,-4,0,0) ; C=(1,2,3,4) (b) A = (6,0,0,0); B=(1,0,2,0) ; C=(4,1,9,2)
Q.7	Write out the proof of Cauchy-Schwarz inequality and triangle inequality.
Q.8	For the given sets of vectors, find the dimension of the space spanned by them and a basis for this space: (a) (1,-1,0,0); (0,-2,5,1); (1,-3,5,1); (2,-4,5,1) (b) (0,10,-1,1,10); (2,-2,-4,0,-3); (4,2,0,4,5); (3,2,0,3,4); (5,-4,5,6,2)
Q.9	Let each of the following matrices represent an active transformation of vectors in (x,y) plane. Show that the matrix is orthogonal, find its determinant, and find the rotation angle or find the line of reflection: (a) $\frac{1}{5}\begin{pmatrix}3 & 4\\ 4 & -3\end{pmatrix}$ (b) $\frac{1}{\sqrt{2}}\begin{pmatrix}-1 & -1\\ 1 & -1\end{pmatrix}$ (c) $\begin{pmatrix}0 & -1\\ -1 & 0\end{pmatrix}$ (d) $\frac{1}{3}\begin{pmatrix}-1 & 2\sqrt{2}\\ 2\sqrt{2} & 1\end{pmatrix}$
Q.10	Verify that each of the following matrices is Hermitian. Find its eigenvalues and eigenvectors, write a unitary matrix U which diagonalizes H by a similarity transformation and show that $U^{-1}HU$ is the diagonal matrix of eigenvalues: (a) $\begin{pmatrix}-2 & 3+4i\\ 3-4i & -2\end{pmatrix}$ (b) $\begin{pmatrix}1 & 2i\\ -2i & -2\end{pmatrix}$

Course Coordinator: Dr. Poorva Singh